Inflation from Geometrical Tachyons

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Abstract

We propose an alternative formulation of tachyon inflation using the geometrical tachyon arising from the time dependent motion of a BPS D3-brane in the background geometry due to k parallel NS5-branes arranged around a ring of radius R. Due to the fact that the mass of this geometrical tachyon field is $\sqrt{2/k}$ times smaller than the corresponding open-string tachyon mass, we find that the slow roll conditions for inflation and the number of e-foldings can be satisfied in a manner that is consistent with an effective 4-dimensional model and with a perturbative string coupling. We also show that the metric perturbations produced at the end of inflation can be sufficiently small and do not lead to the inconsistencies that plague the open string tachyon models. Finally we argue for the existence of a minimum of the geometrical tachyon potential which could give rise to a traditional reheating mechanism.

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1 Introduction.

Despite making many promising advances toward a complete theory of quantum gravity, string/M theory has perhaps not made similar progress in the area of inflationary cosmology where there is an abundance of 4D theoretical models and experimental data [1, 5]. The main focus has effectively been on two types of models. The first is where there is some kind of spontaneous compactification of the extra dimensions of spacetime, leaving us with our familiar four dimensional universe and the standard model. The second model is that the universe itself is located on the world volume of a 3-brane perhaps in a higher dimensional bulk, with the standard model living on the world-volume. Both approaches have proven to be illuminating, yet the problem of inflation in string theory models still proves elusive.

One of the simplest proposals for inflation in a string theory context, has been that of tachyonic inflation [10], where the tachyon plays the role of the inflaton. In the past few years there has been great progress in understanding the nature and role of tachyons in string theory [8], in particular it has emerged that many of the features of the tachyon can be captured surprisingly well by an effective DBI action [17]. Tachyons arise in several contexts in the theory, most notably in the latter stages of brane-antibrane annihilation [1] but also as open string degrees of freedom on a Non-BPS brane [8]. This has stimulated several papers on tachyon inflation and cosmology [11, 12, 13, 14, 15].

However there are several problems associated with open-string tachyon cosmology [4, 16] which appears to cast doubt over the tachyon's role in inflation. The first, and most serious, is that the potential for the field is a runaway exponential, tending to its asymptotic minimum at $T=\pm\infty$.

Thus, not only is this far too steep to generate the required number of e-foldings but there is no minimum for the tachyon to oscillate around and generate reheating [3, 29]. The second problem is that at the start of inflation the de Sitter radius of the universe is actually smaller than the string length and thus an effective theoretical description breaks down, a consequence of this is that the string coupling is large in this region and so perturbative analysis cannot be used. There is also the additional problem of the tachyonic energy, which dominates during inflation and therefore dominates for all time, although arguably this is not as problematic as the other two objections. Whilst there have been several ingenious attempts to bypass these problems, most notably [11, 13, 14, 23], it seems unlikely that the open-string tachyon could be responsible for inflation, although it may still play a role as dark matter fluid [8] or as part of a pre-inflationary phase [3, 5].

Recent work on time dependent solutions in the linear dilaton background of coincident NS5-branes, has shed new light on a possible geometrical description of the open string tachyon [19, 20, 21]. In particular, it is conjectured that radion fields on a probe Dp-brane become tachyonic when the probe moves in a bounded, compact space. In addition, the mass scale of this geometrical tachyonic mode is substantially smaller than that of the open string tachyon, and therefore may resolve some of the problems associated with inflation.

In this note we will examine the effective action for a D3-brane with a geometrical

³Note however that in the case of non-BPS branes in superstring theory, there is evidence that the tachyon potential for static fields develops localized minima at finite values of T, see e.g. [9]

tachyon on its world volume, described by a cosine potential [21]. The cosine potential arises naturally in the context of 'Natural Inflation' [7] due to the creation of pseudo Nambu-Goldstone bosons arising from symmetry breaking, whereas in our case the cosine potential arises due to the geometry of the background brane configuration. It is tempting to identify parameters in the geometrical picture with that of Natural Inflation, however the non-linear form of the tachyon effective action makes this difficult. We will see how our theory fits in with the inflationary paradigm, and whether conventional reheating is possible. We begin by reviewing the construction of the geometrical tachyon solution and the basics of tachyon cosmology. We will then check the consistency of the theory with regards to the slow roll and e-folding approximations before attempting to calculate various perturbation amplitudes and provide a discussion of reheating. We close with some remarks and possible implications for future work.

2 String Background.

We begin this note with a brief description of the string theory background associated with our model. We will consider the NS5-brane background of type II string theory, where we have k parallel but static fivebranes localised on a circle of radius R. The branes will be assumed to be unresolvable for the moment, which can be interpreted as a smearing of the charge of k branes around a ring configuration. The background solutions for these fivebranes are given by the CHS solutions [24]

$$ds^{2} = \eta_{\mu\nu}dx^{\mu}dx^{\nu} + H(x^{n})dx^{m}dx^{m}$$

$$e^{2(\phi-\phi_{0})} = H(x^{n})$$

$$H_{mnp} = -\varepsilon_{mnp}^{q}\partial_{q}\phi,$$
(2.1)

where μ, ν parameterize the directions parallel to the fivebranes and m, n are the transverse directions and ϕ is the dilaton. $H(x^n)$ is the harmonic function which describes the orientation of the fivebranes in the transverse space, which in our simplistic case ⁴ is given by [25]

$$H = 1 + \frac{kl_s^2}{\sqrt{(R^2 + \rho^2 + \sigma^2)^2 - 4R^2\rho^2}}.$$
 (2.2)

In the above solution we have switched to polar coordinates

$$x^{6} = \rho \cos(\theta), \qquad x^{7} = \rho \sin(\theta)$$

$$x^{8} = \sigma \cos(\psi), \qquad x^{9} = \sigma \sin(\psi),$$

$$(2.3)$$

and now the harmonic function describes a ring oriented in the $x^6 - x^7$ plane and we have an $SO(2) \times SO(2)$ symmetry in the transverse space. In our analysis, l_s is the string length and we will be working entirely in string frame. Now the above ring configuration is supersymmetric in 10D, however the introduction of a probe Dp-brane breaks this supersymmetry entirely and the probe brane will experience a gravitational force pulling it toward the fivebranes. To avoid further complications we will always assume that $3 \le p \le 5$, since for

⁴The exact form of the harmonic potential for k NS5 branes arranged around a ring, as computed in [25], is rather more complicated than the expression in (2.2). The latter form emerges as an approximation valid for distances $r >> 2\pi R/k$ i.e. effectively a large k approximation. In this limit the NS5-branes appear as smeared around the ring. [21]

p < 3 there is a divergence due to the emission of closed string modes which will render the classical theory useless [22]. Note that we are also able to switch between IIA and IIB theory because the NS5 brane background in insensitive to T-duality, as the harmonic function couples only to the transverse parts of the metric. We now insert a probe Dp-brane in this background whose low energy effective action is the DBI action, which we write as

$$S = -\tau_p \int d^{p+1} \zeta e^{-(\phi - \phi_0)} \sqrt{-\det(G_{\mu\nu} + B_{\mu\nu} + \lambda F_{\mu\nu})}, \qquad (2.4)$$

where both $G_{\mu\nu}$ and $B_{\mu\nu}$ are the pullbacks of the space-time tensors to the brane, $\lambda=2\pi l_s^2$ is the usual coupling for the open string modes, $F_{\mu\nu}$ is the field strength of the U(1) gauge field on the world volume, and τ_p is the tension of the brane. We will assume that the transverse scalars are time dependent only, and set the gauge field and Kalb-Ramond field to zero for simplicity. Upon insertion of the NS5-brane background solution, we see the action in static gauge reduces to the simple form

$$S = -\tau_p \int d^{p+1} \zeta \sqrt{H^{-1} - \dot{X}^2}, \tag{2.5}$$

with X^m parameterizing the transverse scalar fields. In order to find our geometrical tachyon, we consider motion of the probe brane in the plane of the ring (i.e. $\sigma = 0$ and in the interior of the ring and map the above action to a form that is familiar from the non-BPS action for open-string tachyons. The result is that we have [17, 20, 21]

$$S = -\int d^{p+1}\zeta V(T)\sqrt{1 - \dot{T}^2},$$
(2.6)

where the potential is given by

$$V(T) = \tau_p^u \cos\left(\frac{T}{\sqrt{kl_s^2}}\right)$$

$$\tau_p^u = \frac{\tau_p R}{\sqrt{kl_s^2}},$$
(2.7)

and the tachyon can be expressed as a function of the coordinate ρ as

$$T(\rho) = \sqrt{kl_s^2} \arcsin(\rho/R).$$
 (2.8)

In obtaining the above, we have used the throat approximation for the harmonic function, which means neglecting the factor of unity in (2.2). This may not be necessary, but it does allow an exact expression to be obtained. Under this assumption, we see that taking $\rho = \sigma = 0$ in (2.2) (i.e. the centre of the ring) requires that $\sqrt{kl_s} >> R$, which is the first constraint we find on the parameters k, l_s and R. Later on we will use numerical techniques to arrive at a form of the potential V(T) which will use the exact form of the harmonic function as calculated in [25]. In principle we can then relax the throat approximation which leads to the cosine potential in (2.7) so that the previous inequality may not be needed.

To avoid confusion with the open-string tachyon, from now on (unless otherwise stated) we will use tachyon to refer to the geometrical tachyon in (2.6).

We see that the tachyon potential is symmetric about the origin, which arises as a consequence of the background geometry. It should be noted that this mapping is non-trivial

in the sense that we began by probing a gravitational background, and have ended up with a solution in flat Minkowski space. This tells us that there are two equivalent ways of visualizing the theory. Firstly there is the bulk viewpoint, where there is actually a ring of NS5-branes and the solitary probe brane universe moving in the throat geometry. Alternatively, we could view the problem as a single brane moving in flat space-time with a highly non-trivial field condensing on its world volume. In what follows we will find it useful to switch between these two pictures in order to better understand the physics. In fact the bulk viewpoint is even more complicated as we know [20, 21] that the tachyon field has a geometrical interpretation as a BPS brane in a confined, bounded space, but we could also describe it as a non-BPS brane which has a soliton kink stretched across the interior of the ring [21].

Clearly we see that the geometrical tachyon varies between $T=\pm \pi \sqrt{k l_s^2}/2$ in contrast to the usual open string tachyon which is valued between $\pm \infty$. This is due to the probe brane being confined *inside* the ring. Expanding the potential about the unstable vacuum yields a tachyonic mass of $M_T^2=-1/k l_s^2$. For sufficiently large k this can be made much smaller than the open string tachyonic mass $M^2=-1/2$ (in units where the string length is unity). It is this different mass scale and profile of the potential which suggest that the geometrical tachyon may be useful in describing inflation on a D3-brane. The energy momentum tensor can be calculated in the usual way, and has non-zero components

$$T_{00} = \frac{V(T)}{\sqrt{1 - \dot{T}^2}}$$

$$T_{ij} = -\delta_{ij}V(T)\sqrt{1 - \dot{T}^2}$$
(2.9)

from which we see that the pressure of the tachyon fluid tends to zero as the tachyon rolls toward the zero of V(T).

In [21] we found that there was also probe brane motion through the ring in the $x^8 - x^9$ plane. Although this did not lead to the creation of a geometrical tachyon, one could imagine a scenario where the probe oscillates in this direction through the ring, radiating energy as it does so. Eventually the probe would settle at the origin, which is an unstable point in the ring plane corresponding to T = 0, and we then recover our geometrical tachyon solution.

3 Tachyon Cosmology.

In a cosmological context, the condensing tachyon will generate a gravitational field on the probe D3-brane and therefore we must include this minimal coupling in the action. We will also assume that there is no coupling to any other string mode in order to keep the analysis simple, however we should be aware that there is no reason why other modes should not be included [18]. Our Lagrangian density can thus be written

$$\mathcal{L} = \sqrt{-g} \left(\frac{R}{16\pi G} - V(T) \sqrt{1 + g^{\mu\nu} \partial_{\mu} T \partial_{\nu} T} \right), \tag{3.1}$$

where $g^{\mu\nu}$ is the metric and R is the usual scalar curvature. For simplicity we will assume that there is a FLRW metric of the form

$$ds^2 = -dt^2 + a(t)^2 dx_i^2, (3.2)$$

with i running over the spatial directions. We have implicitly assumed here that we have a flat universe, which is acceptable because any curvature is negligible in the very early stages of the universe. The effect of the scale factor is to modify the energy density, u for the flat background such that it is no longer conserved in time [10], instead we find

$$u = \frac{a^3 V(T)}{\sqrt{1 - \dot{T}^2}},\tag{3.3}$$

which prevents us from obtaining an exact solution for the tachyon in the presence of the gravitational field in the usual manner. From this we can determine the late time behaviour of the tachyon condensate. If we assume that u is constant, then the pressure will vary as $p = -V(T)^2u$ and will tend to zero as V(T) reaches its minimum. Using the standard equation of state $p = \omega u$, we find that $\omega = -(1 - \dot{T}^2)$ which implies $-1 \le \omega \le 0$. From the Lagrangian density, we can also obtain the Friedman and Raychaudhuri equations for the tachyon condensate

$$H^{2} = \left(\frac{\dot{a}}{a}\right)^{2} = \frac{\kappa^{2}V(T)}{3\sqrt{1 - \dot{T}^{2}}} \tag{3.4}$$

$$\frac{\ddot{a}}{a} = \frac{\kappa^2 V(T)}{3\sqrt{1 - \dot{T}^2}} \left(1 - \frac{3\dot{T}^2}{2} \right),$$
 (3.5)

where $\kappa^2 = 8\pi G = M_p^{-2}$, $M_p = 2.2 \times 10^{18} GeV$ and the cosmological constant term is set to zero. There is also a useful relationship between the 4D Planck mass and the string scale obtained via dimensional reduction [2]

$$M_p^2 = \frac{vM_s^2}{g_s^2},\tag{3.6}$$

where $M_s = l_s^{-1}$ is the fundamental string scale and the quantity v is given by

$$v = \frac{(M_s r)^d}{\pi},\tag{3.7}$$

with r, d being the radius and number of compactified dimensions respectively (typically d=6 for the superstring). Note that for our effective theory to hold we require v >> 1. Obviously we are assuming there is some unknown mechanism which stabilizes the compactification manifold, and freezes the moduli so that they do not interfere with our tachyon solution.

The evolution of the universe is effectively determined by the Raychaudhuri equation which shows that inflation will cease when $\dot{T}^2=2/3$ and the universe will then decelerate as the tachyon velocity increases. Upon variation of the action, we find the equation of motion for the tachyon field can be written

$$\frac{V(T)\ddot{T}}{1 - \dot{T}^2} + 3HV(T)\dot{T} + V'(T) = 0,$$
(3.8)

where a prime denotes differentiation with respect to T, and H is the Hubble parameter. Note that in deriving this equation we must also use the conservation of entropy of the tachyon fluid [10]. We see that $3HV(T)\dot{T}$ acts as a friction term, in much the same way as in standard inflationary models, except that this term may vanish for the open string tachyon as the field rolls to $\pm \infty$ where its potential vanishes. For a scalar field to be a candidate

for the inflaton it must satisfy the usual slow roll parameters as well as providing enough e-foldings during rolling. The tachyon is no exception [6], and so we use the conventions employed in [14, 26] to write the slow roll parameters

$$\epsilon(T) = \frac{2}{3} \left(\frac{H'(T)}{H^2(T)}\right)^2$$

$$\eta(T) = \frac{1}{3} \left(\frac{H''(T)}{H^3(T)}\right).$$
(3.9)

Where we assume that the acceleration of the tachyon is negligible, and require that $\epsilon << 1$ and $|\eta| << 1$ in order to generate inflation. There appears to be little or no consensus on the correct slow roll equations to use for the tachyon, however we see that the same general behaviour is obtained even if we use the conventions in [12] or [15]. The number of e-folds produced between T_o and T_e is given by

$$N(T_o, T_e) = \int_{T_o}^{T_e} dT \frac{H}{\dot{T}}, \qquad (3.10)$$

which must satisfy $N \sim 60$ to agree with observational data [1]. T_o and T_e are two arbitrary points on the potential where the slow roll conditions are still satisfied.

4 Geometrical tachyon inflation.

One of the many problems associated with tachyon inflation is that the mass scale of the open string tachyon is simply too large. However we have seen that this is not necessarily the case where we have a geometrical tachyon, and so we may enquire whether inflation is possible in this instance. We first consider the geometrical tachyon starting very close to the top of its potential with a small initial velocity to ensure that it will roll. Note that if T=0 then spontaneous symmetry breaking will cause the universe to fragment into small domains which will each have differing values of the tachyon field. Inflation can occur only if $H^2 >> |M_T^2|$ near the top of the potential, which translates into the constraint

$$\frac{\tau_3 R}{3M_p^2} >> \frac{1}{\sqrt{kl_s^2}}.$$
 (4.1)

Where τ_3 is the tension of a stable D3-brane. Since we are considering the large k limit, the RHS is very small and so we find that this condition is satisfied. ⁵ Furthermore this also suggests that the effective theory for geometrical tachyons is valid because we can clearly see that

$$(kl_s^2)^{1/2} >> H^{-1}, (4.2)$$

and so the de-Sitter horizon may be larger than the string length for large k. This is in contrast to the open string scenario where we find that the horizon is smaller than the string length, and thus should not be described by an effective theory.

In order to check that this is correct, we must try and get a handle on the size of the string coupling. In the open string tachyon case we find that in order to satisfy $H^2 >> |M_T^2|$ at the top of the potential, we have the constraint

$$g_s >> 260v. \tag{4.3}$$

 $^{^{5}}$ In the finite k case we will have to be careful to ensure that this constraint is fulfilled.

As already mentioned, the effective theory is only valid for v >> 1, which implies that we are in the strong coupling regime and therefore an effective theory may not yield reliable results. In contrast, a similar calculation for the geometrical tachyon implies

$$g_s >> \frac{24\pi^3 v}{\sqrt{k}RM_s} \approx 744 \frac{v}{\sqrt{k}RM_s},\tag{4.4}$$

thus by fixing appropriate values for k and R we may ensure that v >> 1 and also that $g_s << 1$. Earlier we saw the throat condition $\sqrt{k}l_s >> R$ which means that $\sqrt{k}RM_s << k$. Thus, in fact it is large values for k that will essentially allow a satisfactory solution to (4.4). For example, assuming v=10 a value of $k\approx 10^5$ would allow for perturbative g_s to solve (4.4). Relaxing the throat approximation may allow much smaller values of k. This is interesting because we see that the weak coupling arises entirely due to a parameters describing the background brane solution in the bulk picture. In contrast, there is not a clear explanation to account for the origin of the weak self-coupling of the inflaton in standard single scalar field [6] inflation.

Despite this apparent success we may be concerned that the effective theory may still not be a valid description at the top of the potential [4]. In order to check this we should compare the effective tension of the unstable brane to the Planck scale. After some algebra, and using the equation for weak coupling we find

$$\frac{\tau_3^u}{M_p^4} \sim \frac{3}{k^2} \left(\frac{24\pi^3}{RM_s}\right)^2 v. \tag{4.5}$$

Again we see that for a certain range of background parameters (and assuming $RM_s >> 1$), the effective tension need not be Super-Planckian and therefore the DBI can still be a good approximation to 4D gravity.

As there is an obvious similarity with Natural Inflation [7] we could demand that the height of the potential to be of the order of M_{GUT}^4 (where $M_{GUT} \sim 10^{16}$ GeV) in order to generate inflation, however we will try to keep this arbitrary for the moment. Using the potential, we immediately see that the slow roll conditions for our geometrical tachyon can be written as

$$\epsilon = \frac{M_p^2}{2\tau_3 R \sqrt{kl_s^2}} \frac{\tan^2(T/\sqrt{kl_s^2})}{\cos(T/\sqrt{kl_s^2})}$$
(4.6)

$$\eta = \frac{-M_p^2}{4\tau_3 R \sqrt{kl_s^2}} \frac{\left(1 + \cos^2(T/\sqrt{kl_s^2})\right)}{\cos^3(T/\sqrt{kl_s^2})}.$$
 (4.7)

Slow roll will only be a valid approximation when the tachyon is near the top of its potential and thus we may effectively neglect the contribution from the trigonometric functions as they will only be terms of $\mathcal{O}(1)$. In fact for small T we see that ϵ is already extremely small. Dropping all numerical factors of order unity gives us the primary constraint for slow roll;

$$M_p^2 << \tau_3^u R \sqrt{k l_s^2},\tag{4.8}$$

which must be satisfied by both equations. Given that the reduced Planck mass in string theory is typically of the order of 2.4×10^{18} GeV, this means that k and R/l_s must be large. Generally the slow roll conditions will be satisfied due to the mass scale of the geometrical

tachyon, for large k. In the open string models, the larger mass implies that the tachyon may only have been involved in some pre-inflationary phase. Of course, the analysis in both cases is classical and quantum corrections may well prove to be important is determining the exact behaviour near the top of the potential.

We can estimate the number of e-foldings using (3.10), however this turns out to be sensitive to the value of the tachyon velocity near the top of the potential. To remedy this we use the equations of motion and the slow roll approximation, which allows us to re-write this equation in terms of the potential and its derivative. In fact this is the method most commonly used in standard inflationary analysis. After some algebra we obtain

$$N_e = -3 \int dT \frac{H^2 V(T)}{V'(T)}$$

$$= \frac{\tau_3 R \sqrt{k l_s^2}}{M_p^2} \left\{ \cos \left(\frac{T_e}{\sqrt{k l_s^2}} \right) - \cos \left(\frac{T_o}{\sqrt{k l_s^2}} \right) + \ln \left(\frac{\tan(T_e/2\sqrt{k l_s^2})}{\tan(T_o/2\sqrt{k l_s^2})} \right) \right\}$$

$$(4.9)$$

Using the constraint from the slow-roll equations we see that the leading term must be large. If we demand that T_o and T_e are reasonably close, then the contribution from the other terms will be small, and so the number of e-foldings will depend on the ratio

$$\nu = \frac{\tau_3 R \sqrt{k l_s^2}}{M_p^2},\tag{4.10}$$

where $\nu \geq 60$ in order for there to be enough inflation. However if we dont impose this restriction, but allow inflation to begin near the top of the potential and end near the bottom, then there can be significant contribution to the number of e-foldings from the additional terms. This has the effect of reducing the value of ν - however it must still satisfy the slow roll constraint of being larger that unity. We can write the unstable brane tension in terms of the string coupling, string mass scale and the parameter ν , thus we have the height of the potential given by

$$M_{infl}^4 = \frac{M_p^2 M_s^2 \nu}{k},\tag{4.11}$$

which defines our effective inflation scale M_{infl} . The exact value of M_s depends on the particular string model but it is usually assumed to lie in the range 1 Tev - 10^{16} GeV. So as an example, if $\nu \sim 60$, $M_s \sim 10^{16}$ GeV and $k \sim 10^5$ we find $M_{infl} \sim 10^{16}$ GeV.

4.1 Numerical Analysis.

We can also check the consistency of our analytic solutions by numerically solving for the Hubble parameter. We can write the Hubble equation as a function of T rather than time (since the tachyon field is monotonic with respect to time - at least initially), and then using the Friedmann equation we obtain the following first order differential equation [15]

$$H^{2}(T) - \frac{9}{4}H^{4}(T) + \frac{1}{4M_{p}^{4}}V(T)^{2} = 0, \tag{4.12}$$

where a prime denotes differentiation with respect to T. Solving this for the Hubble term gives us a constraint on the velocity of the tachyon field

$$\dot{T}^2 = 1 - \left(\frac{V(T)}{3M_p^2 H(T)^2}\right)^2. \tag{4.13}$$

It will be convenient to work with dimensionless variables in our numerical analysis, so we define the dimensionless tachyon field and Hubble parameter as follows,

$$y = \frac{T}{\sqrt{kl_s^2}}, \quad h(y) = \sqrt{kl_s^2}H(y).$$
 (4.14)

We can solve (4.12) to obtain h(y) and then substitute this into (4.13) to determine the velocity of the tachyon field. We choose the initial velocity of the field to be zero, and the initial value of T_o to be very small. As in [15], the general behaviour is dependent upon the dimensionless ratio X_0 , where $X_0^2 = \nu$. Some results are plotted in Figure 1. We find that the velocity (strictly speaking this is the square of the velocity) of the tachyon field is very small over a large range, only becoming large as it nears the bottom of the potential. In inflationary terms this implies that universe will be inflating for almost the entire duration of the rolling of the field. For increasing values of X_0 , inflation ends at larger values of T. However even for the case of $X_0 = 2$, which barely satisfies the slow roll constraints, we expect inflation to end reasonably close to the bottom of the potential. We can also make

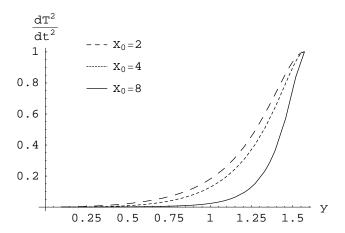


Figure 1: Velocity of tachyon field for differing values of X_0 , with an initial velocity of zero.

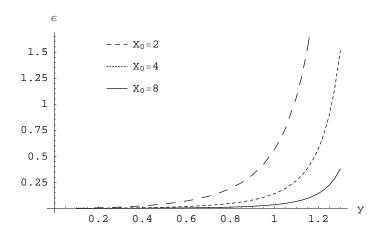


Figure 2: Value of the slow-roll parameter ϵ for various values of X_0

a numerical check on the smallness of the slow roll parameters ϵ , η (see figs 2 and 3). Using our numerical solution for h we can also determine the amount of e-foldings during inflation

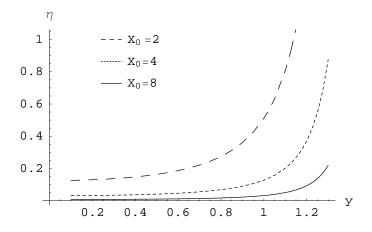


Figure 3: Value of the slow-roll parameter $|\eta|$ for various values of X_0

via (4.9). It turns out that in order to generate at least 60 e-foldings we only need to take $\nu \sim 30$

Finally, we can also use numerical techniques to try and reconstruct the tachyon potential by using the full form of the ring harmonic function as derived in [25] without assuming the approximation that lead to the cosine potential (2.7). Recall that this approximation was that the NS_5 branes were unresolvable as point sources arranged uniformly around the ring. As our tachyon field rolls from near the top of the cosine potential down towards the value $T/\sqrt{k}l_s = \pi/2$, the geometric picture of this process is that we start from near the centre of the ring at $\rho = 0$ and move towards the ring located at $\rho = R$. As $T/\sqrt{kl_s}$ nears $\pi/2$, even for large k, the approximation of a continuous distribution of NS5 branes around the ring will break down and individual sources will be resolvable. It is at this point that we expect the true potential V(T) to deviate from the cosine form. Fig 4 shows the shape of the potential one obtains for the case k = 1000, by numerically implementing the tachyon map discussed earlier, using the exact form of the ring harmonic function. In this plot we have chosen the angular variable θ that appears in the exact form of the harmonic function to be fixed at $\pi/2k$ for simplicity. Details of the harmonic function relevant to fully resolvable NS5 branes are given later on in Section 5. What is perhaps most apparent about this potential is the existence of a minimum very close to the ring location. It also turns out that our previous cosine potential is an excellent approximation to this numerical plot for values of T to the left of the minimum. Later on in section 5, we shall see how analytic methods can be used to verify the existence of this minimum.

4.2 Perturbations.

So far, so good, but we must also consider the perturbation fluctuations generated at the end of inflation. One of the generic difficulties associated with open-string tachyonic inflation is the fact that the tension of the D3-brane must be significantly larger than the Planck mass. This indicates that the effective action cannot adequately describe 4D gravity, as it will have metric fluctuations that are always too large [4]. This is not the case for our geometrical tachyon as we seen there are additional scales in the theory which can reduce the overall effect of these fluctuations. There are two main perturbations to consider, the

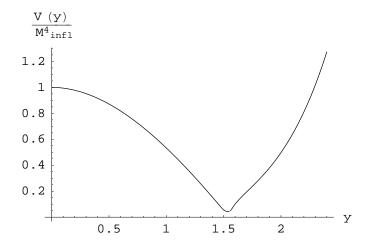


Figure 4: Profile of the tachyon potential taking k=1000. The solutions from each region are matched onto each other at $T=\pi/2$ in dimensionless units.

scalar, and the gravitational (tensor) ones which we will denote by $\mathcal{P}_{\mathcal{T}}$ and $\mathcal{P}_{\mathcal{G}}$ respectively. (Strictly speaking, \mathcal{P} corresponds to the amplitude of the perturbation). Constraints from observational data imply the relation ⁶

$$|\mathcal{P}_{\mathcal{T}}| + |\mathcal{P}_{\mathcal{G}}| < 10^{-5}. (4.15)$$

During inflation, gravitational waves are produced whose amplitude is given by $\mathcal{P}_{\mathcal{G}} \sim \frac{H}{M_p}$, but observational data of the anisotropy of the CMB [4, 14, 15] implies that at the end of inflation

$$\frac{H_{end}}{M_p} \le 3.6 \times 10^{-5},\tag{4.16}$$

and we must ensure that this condition is consistently satisfied in order for us to consider the geometrical tachyon as a possible candidate for the inflaton. In order to verify this we will first consider the scalar perturbation and use the solution from that to determine our mass scales for the metric perturbations, as it is generally more important to see whether the tachyon action allows for small metric fluctuations. For simplicity we will assume that inflation ends when the following constraint is satisfied [4]

$$H \sim |M_T| \sim \frac{M_s}{\sqrt{k}},$$
 (4.17)

and we will also assume that the tachyon velocity at this time is given by $\dot{T} = \sqrt{2/3}$. The scalar perturbations are determined in the usual manner using

$$\left|\frac{\delta\rho}{\rho}\right| \sim \frac{H\delta T}{\dot{T}},$$
 (4.18)

where δT satisfies the following constraint near the top of the potential [15, 27]

$$\delta T \sim \frac{H^2}{2\pi\sqrt{V(T)}}. (4.19)$$

Combining the last two equations we write the amplitude for the scalar perturbation as

$$\mathcal{P}_T \sim \frac{H^2}{2\pi \dot{T} \sqrt{V(T)}} \le 10^{-5}.$$
 (4.20)

⁶Thanks to D. J. Mulryne for pointing this out.

(We should actually calculate the values of H and \dot{T} during inflation in order to determine the ratio of the perturbations, however since we expect T to be a slowly varying field (4.20) should remain constant over a large range of wavelengths [5].) If we assume that T is small then the cosine part of the potential is close to unity, and upon substitution of the Hubble term we find

$$\mathcal{P}_T \sim \frac{M_s}{M_p} \sqrt{\frac{3}{8\pi^2 k\nu}} \le 10^{-5},$$
 (4.21)

We can use this to determine a constraint upon the string scale/Planck scale ratio as follows

$$\frac{M_s}{M_p} \le \sqrt{\frac{8\pi^2 k\nu}{3}} \times 10^{-5}. (4.22)$$

As an example, for $k \sim 10^3$ and $\nu \sim 28$ (4.22) implies $M_s \leq 10^{16} GeV$

Solving the equation for the metric perturbation leaves us with

$$\mathcal{P}_G \sim \frac{H}{M_p} \sim \frac{M_s}{M_p \sqrt{k}} \le 3.6 \times 10^{-5}$$
 (4.23)

which is explicitly dependent upon this ratio. We can establish the absolute upper bound on the perturbation using (4.22)

$$\mathcal{P}_G \le 2\pi \times 10^{-5} \sqrt{\frac{2\nu}{3}}.$$
 (4.24)

If ν is $\mathcal{O}(30)$ then this implies the maximum perturbation will be of the order of 10^{-4} which is slightly too large. However in general we may expect that the metric perturbations will be acceptably small by assuming a smaller string scale than the one that saturates (4.22) for given k. This is encouraging since the open string tachyon always admits large metric fluctuations, and therefore cannot be responsible for the last 60 e-foldings of inflation [3]. In our case these fluctuations can be suppressed when k is sufficiently large, and we can find inflationary behaviour leading to the correct amount of structure formation.

5 Reheating.

Perhaps the most problematic aspect of tachyon inflation is the shape of the potential itself. The open string tachyon potential is exponentially decaying at large field values with its minimum at asymptotic infinity. Thus even if it were possible to satisfy all the inflationary conditions, the lack of minimum effectively kills the model as there will be no reheating [3] in the classical sense. (As mentioned previously, gravitational reheating is far too weak in these models to account for the particle abundance we see today.) It is possible to obtain reheating if the tachyon is coupled to several gauge fields [11], and is a direction that certainly needs further consideration. It is also certainly possible that the potential vanishes for finite T, leading to small oscillations about the minimum [15] which could provide a mechanism for reheating. In any case, the issue does not seem to be resolved in a satisfactory manner.

Our geometrical tachyon is no exception to these criticisms. Although the minimum is not at infinity, the effective theory breaks down when the tachyon rolls to its maximum

value and we are unable to proceed. In the 10D gravitational picture this is due to the probe brane hitting the ring of smeared fivebranes. However even with the simple form of the DBI action in this instance, we see that outside the ring the potential is approximately exponential [21] and it is suggestive that it may somehow smoothly map onto the cosine at $\rho = R$. One may well enquire what happens if we consider a case where the fivebranes are not smeared around the ring, rather that they appear resolved thus allowing a probe brane to pass between them. In this case, we would not expect the effective DBI action to break down and we can find corrections to the truncated cosine potential and thus obtain a minimum. This is exactly what we found following the numerical analysis in section 4. Let us now see how the existence of such a minimum can be seen analytically. In order to proceed, we refer the reader back to the full harmonic function describing k branes at arbitrary points on the circle [25], with the interbrane distance, x, given by

$$x = \frac{2\pi R}{k}. (5.1)$$

The full form of the function in the throat region is given by

$$H \sim \frac{kl_s^2}{2R\rho\sinh(y)} \frac{\sinh(ky)}{(\cosh(ky) - \cos(k\theta))},\tag{5.2}$$

where ρ, θ parameterize the coordinates in the ring plane, and the factor y is given by

$$\cosh(y) = \frac{R^2 + \rho^2}{2R\rho}.\tag{5.3}$$

We clearly see that as $k \to \infty$ we recover the expression for the smeared harmonic function which we used in the previous sections to derive the tachyon potential. Furthermore we see that when $\rho = R$ the function reduces to

$$H \sim \frac{k^2 l_s^2}{2R^2} \frac{1}{(1 - \cos(k\theta))}$$
 (5.4)

which is clearly finite provided that $\theta \neq 2n\pi/k$, which are the locations of the NS5 branes. In order to look for a minima we must expand about the point $\rho = R$ using $\rho = R + \xi$, where ξ is a small parameter which can be positive or negative. Using the expansion properties of hyperbolic functions we power expand the harmonic function for an arbitrary fixed angle θ , and we find to leading order⁷

$$H \sim \frac{k^2 l_s^2}{2R^2} \frac{1}{1 - \cos(k\theta)} \left(1 - \left| \frac{\xi}{R} \right| + \left(\frac{5}{6} - k^2 \frac{2 + \cos(k\theta)}{6(1 - \cos(k\theta))} \right) \frac{\xi^2}{R^2} + \dots \right), \tag{5.5}$$

where we have used the fact that $k\xi \ll R$ and have neglected any higher order correction terms. Note that the inter brane distance is given by $2\pi R/k$ and so our expansion will only be valid for distances much smaller than the brane separation. Of course we must be careful not to take k to be too small since our effective action for the geometrical tachyon will be invalid. We can clearly see that if the trajectory is at an angle $\theta = (2n + 1)\pi/2k$, then the harmonic function will reduce to the form (again to leading order in large k)

$$H \sim \frac{k^2 l_s^2}{2R^2} \left(1 - \frac{k^2 \xi^2}{3R^2} + \dots \right)$$
 (5.6)

⁷Thanks to Shinji Tsujikawa for pointing out an algebraic error in a previous draft of this note.

We now perform the tachyon map to determine the value of the tachyon as a function of ξ . Note that we expect this 'tachyonic' field to have positive mass squared since it is near the minimum of its potential. Up to constants we find that

$$T(\xi) \sim \sqrt{\frac{k^2 l_s^2}{2(1 - \cos(k\theta))}} \left(\frac{\xi}{R} - \frac{\xi^2}{2R^2} + \dots\right)$$
 (5.7)

If we assume that the ξ^2 term is negligible then we can invert our solution and calculate the perturbed tachyon potential. Note that keeping higher order terms here does not lead to a simple analytic solution, and so we would hope that a numerical analysis would be more appropriate. After some manipulation we find

$$V(T) \sim \frac{\tau_3}{kl_s} \left(2R^2 (1 - \cos(k\theta)) \right)^{1/2} \left[1 + \frac{T}{2kl_s} \sqrt{2(1 - \cos(k\theta))} + T^2 \left(\frac{2 + \cos(k\theta)}{6l_s^2} - \frac{(1 - \cos(k\theta))}{12k^2l_s^2} \right) + \ldots \right]$$
(5.8)

which shows that the potential is approximately linear around the minimum as it interpolates between the cosine and the exponential functions, however this linear term is suppressed by a factor of 1/k and we would expect it be negligible in the large k limit, thus we can see that there is an approximately quadratic minimum. We see that the minimum of the potential in the tachyonic direction will be

$$V(T(\xi = 0)) = \frac{\tau_3 R}{k l_s} \sqrt{2(1 - \cos(k\theta))}$$
 (5.9)

which can obviously be made small in the large k limit, and will clearly go to zero when the D3-brane trajectory is such that it hits one of the NS5-branes. The local maximum will occur at the bisection angle $\theta=\pi/k$, which we suspect will be an unstable point. All this fits nicely with our earlier numerical analysis. Figure 4 in section 4 showed the result of numerical methods used to plot the potential using the exact form of the ring harmonic function. Numerical solutions to the tachyon map inside and outside the ring were matched together to obtain this plot. The minimum can be seen to be quadratic for small distances before mapping onto an exponential function outside the ring as expected from [21]. This is because the numerical analysis includes all the higher order correction terms, which produces a curved potential at the minimum.

The condensing tachyon field may oscillate about the minimum of this potential, assuming that the energy of the tachyon is such that it will not overshoot and return back up the potential toward T=0. This assumption seems to be valid because as we have just seen the potential no longer has to vanish at $\rho=R$, so the friction term in (3.8) will not vanish as the tachyon condenses. However in order for reheating to occur we must ensure that this term sufficiently damps the motion, confining the field to very small oscillations about this minimum.

From standard inflationary models we know that the oscillations about the minimum can be thought of as being a condensate of zero momentum particles of $(mass)^2 = V''(T)$. The decay of the oscillations leads to the creation of new fields coupled to the tachyon condensate via the reheating mechanism. The temperature of this reheating can be approximated as the difference between the maximum and minimum of the potential, and so we find

$$T_{RH}^4 \sim M_{infl}^4 \left(1 - \frac{1}{\sqrt{k}} \sqrt{2(1 - \cos(k\theta))} \right)$$
 (5.10)

and so if we assume that the conversion of the tachyon energy is almost perfectly efficient then we will have an upper bound for the reheating temperature given by the effective inflation scale M_{infl} .

We must now consider the more general case where we perturb θ away from its bisection value of π/k . Since we are assuming that the NS5-branes are somehow resolvable, we must also be aware that a single brane does not form an infinite throat [20]. As such, a passing probe brane will feel the gravitational effect of the fivebranes, but because we expect it to be moving relativistically we expect that its trajectory will only suffer a slight deflection as it passes by. In this instance, the perturbed harmonic function at $\rho = R$ reduces to

$$H \sim \frac{k^2 l_s^2}{2R^2} \frac{1}{(1 + \cos(k\delta))},$$
 (5.11)

where δ represents the angular perturbation. Now, we know that the harmonic function becomes singular when our probe brane hits a five brane so the function needs to be minimized to ensure a stable trajectory, This is clearly accomplished by sending $\delta \to 0$. So the value π/k corresponds to an unstable maximum from the viewpoint of the tachyon potential. Of course, we could also see this directly from (5.9) by considering perturbations about the bisection angle. For small angular deflection we may write

$$H \sim \frac{k^2 l_s^2}{4R^2} \left(1 + \frac{k^2 \delta^2}{4} + \dots \right),$$
 (5.12)

and so we see that provided $k\delta << 1$ the trajectory of the probe will be relatively unaffected by the presence of the fivebranes and therefore we may expect that it will pass between them. On the other hand, for larger values of $k\delta$, this will not be true and the probe brane may be pulled into the fivebranes. In any case, we expect that our analysis of the geometrical tachyon will be invalid in this instance.

The analysis will also be true for a D3-brane in a ring D5-brane background using S-duality, the only difference will be to replace

$$kl_s^2 \to 2g_s kl_s^2,\tag{5.13}$$

where g_s is the string coupling and we again consider k branes on the ring. The overall effect of switching to the D5-brane background is to allow for a weaker coupling at the top of the potential. In fact the analogue of (4.4) in this picture becomes

$$g_s >> \left(\frac{24\pi^3 v}{R\sqrt{2k}M_s}\right)^{2/3} \tag{5.14}$$

The situation is made slightly more complicated due to the presence of background RR charge, but this can be neglected when the tachyon is purely time dependent. Thus we would expect similar results to those obtained in the last two sections. Of course, we should remember that fundamental strings can end on the D5-branes and consequently there can be additional open string tachyons in the theory.

6 Discussion

In this note we have examined the cosmological consequences of the rolling geometrical tachyon in the early universe. Because of the different mass scale compared to the open

string tachyon, the geometrical tachyon resolves some of the problems besieging tachyon inflationary models.

The effective theory is self consistent and appears to be valid description of 4D gravity due to the weak string coupling. This weak coupling arises because we can select a specific region of our moduli space associated with the background geometry. Furthermore we have seen that the form of the potential satisfies all the slow roll and e-folding conditions, whilst providing acceptable levels of metric perturbations at the end of inflation. In addition, we have shown that the potential has a minimum which will be approximately quadratic for small perturbations in the tachyon field and may therefore be used to describe traditional reheating [3]. We have not in any way discussed how reheating can occur in this model, but we have attempted to show that the potential may well have a metastable minimum which could allow for the kind of field theoretic inflaton oscillation required for standard reheating.

There are still potential problems associated with this model. Firstly it seems unlikely that we can have an analytic expression for the tachyon valid for any point on the potential due to the complicated nature of the full harmonic function. Thus we have been forced to make approximations or resort to numerical methods. Secondly there is still the issue of fine tuning to deal with because we need specific values of k and R such that a probe brane passes between fivebranes in the bulk picture without causing the tachyon solution to collapse. Furthermore we effectively require the D3-brane to exhibit one dimensional motion, so that it continually passes between the NS5 branes on the ring as it rolls in the minimum of the potential. More general motion would imply that the probe brane will eventually hit one of the source branes and cause the effective theory to break down. Thirdly we must ensure that the probe brane is not too energetic, otherwise it may overshoot the minimum and return to the origin. This requires the friction term to sufficiently damp the motion of the field as it approaches the minimum of the potential. Although we have argued that this may occur it is not clear whether this requires any fine tuning or not. Finally there is the issue of coupling to other string modes, which will be essential in generating the standard model fields after reheating, and have been neglected in this and other notes on tachyon inflation.

A more detailed investigation is required before we can rule out this model, however we have found some promising results that circumvent many of the problems associated with tachyon inflation and it is suggestive that other geometrical tachyon modes may well be viable candidates for the inflaton. It would also be useful to extend this work to include assisted inflation, and perhaps non-commutative geometrical tachyons, for example [28]. In addition, it would be useful to have more understanding of these geometrical tachyon modes and their relationship to the open string tachyon. In particular the case of k=2 is important [20], as it relates observables in Little String Theory (LST) to those in 10D supergravity. Because the geometrical tachyon fields are dependent upon the background brane geometry, we may imagine that all these tachyon solutions correspond to specific points or cross sections of some larger moduli space. It would therefore be useful to consider other background geometries and how they are inter-related to some of the known open string tachyon solutions. We will hopefully return to some of these issues at a future date.

Acknowledgements

⁸ Although there may be some objections to this due to the non-linearity of the tachyonic action [29].

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